## §22. Dynamo in a Precessing Sphere

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**i) Introduction** Despite of the simple motion of the precessing sphere itself, the inner flows are too complex to be fully understood on the basis of analytical theories or experiments. Hence, numerical simulations have been playing an important role to investigate characteristics of flows in a precessing cavity and the associated MHD dynamo. A turbulent ring was discovered numerically in a precessing sphere by Kida and Shimizu [1], along which strong vorticity as well as magnetic flux are generated. (figure 1) In the present study, we focus on the flow structures in a precessing sphere and reveal the mechanism to create the ring.

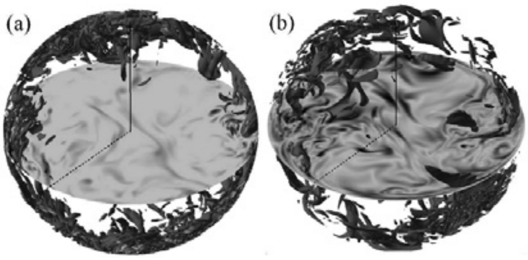


figure 1: Turbulent ring. Isosurfaces of vorticity field (a) and magnetic field (b) are plotted. [1]

**ii) Governing Equations** We consider the MHD dynamo driven by incompressive flows in a precessing sphere with the magnitude of the spin and precession angular velocities being constant in time and two axes being orthogonal. The evolution equations in the sphere for the fluid velocity $\boldsymbol{u}(\boldsymbol{r}, t)$ and the magnetic flux density $\boldsymbol{b}(\boldsymbol{r}, t)$ may be written in the precession frame $(x, y, z)$ which is rotating with the precession angular velocity $\boldsymbol{\Omega}_{\mathrm{p}} = \Omega_{\mathrm{p}}\hat{\boldsymbol{z}}$ , in non-dimensional form, as

$$\nabla \cdot \boldsymbol{u} = 0, \quad \nabla \cdot \boldsymbol{b} = 0,$$

$$\frac{\partial \boldsymbol{u}}{\partial t} = \boldsymbol{u} \times (\nabla \times \boldsymbol{u}) - 2\Gamma \hat{\boldsymbol{z}} \times \boldsymbol{u}$$

$$- \nabla P - \boldsymbol{b} \times (\nabla \times \boldsymbol{b}) + \frac{1}{Re}\nabla^2 \boldsymbol{u},$$

$$\frac{\partial \boldsymbol{b}}{\partial t} = \nabla \times (\boldsymbol{u} \times \boldsymbol{b}) + \frac{1}{Re_m}\nabla^2 \boldsymbol{b},$$

where $P$ is the modified pressure, $\Gamma = \Omega_{\mathrm{p}}/\Omega_{\mathrm{s}}$ the Poincare number, $Re$ the Reynolds number, $Re_m$ the Magnetic Reynolds number, $\Omega_{\mathrm{s}}$ the spin angular velocity taken in the $x$ direction. The outside of the sphere is assumed to be vacuum, where the magnetic flux density $b^{(o)}$ obeys $\nabla \cdot \boldsymbol{b}^{(o)} = 0$ and $\nabla \times \boldsymbol{b}^{(o)} = 0$. These equations are supplemented by (on $r = 1$),

$$\boldsymbol{u} = \hat{\boldsymbol{x}} \times \boldsymbol{r}, \quad \boldsymbol{b} = \boldsymbol{b}^{(o)} \quad (\text{on} \quad r = 1)$$

which are the boundary conditions derived from the assumptions that the flow is non-slip on the boundary and the magnetic permeability of the fluid is equal to that of vacuum. We also assume that $b^{(o)} = \mathbf{0}$ at infinity.

**iii) Laminar and turbulent ring** In the following, we fix the Poincare number at$\Gamma = 0.1$ The flow states change steady, periodic and turbulent as $Re$ increases.[2] When $Re$ and $Re_m$ are large enough, turbulence and turbulent dynamo are sustained in the sphere. One of the most remarkable features of turbulent structure is that there exists a ring-like structure along which turbulence and magnetic field are active.[1] This observation implies that to investigate the creation process of the turbulent ring is the key to understand the sustaining mechanism of turbulent vorticity and magnetic fields.

This ring structure is observed even at lower Reynolds numbers, at which flow is not turbulent but steady or periodic. [3] This fact is verified in figure 2. It is also observed that these rings are inclined slightly from the spin axis, and the angle of inclination seems almost independent of the Reynolds number. The velocity field is similar to that of spin-up from the rest without precession. As the spin axis rotates about the precessing axis, fluid inside the sphere continuously tries to approach the solid-body rotation flow, but this spin-up is never established. This is because it takes the duration of the order of $1/\Omega_s$ to evolve the boundary layer and to start the spin-up; whereas during this period the ring with high angular velocity inclines at an angle of the order $\Omega_p/\Omega_s = \Gamma$. Thus, the laminar ring is likely to be created by this continuous spin-up. As shown in the previous subsections, the laminar ring observed at lower $Re$ is embedded in turbulence sustained at higher $Re$ as a large-scale flow structure. The active regions, i.e. the turbulent ring, of vorticity and magnetic fields may well be created along the laminar ring, since the turbulence and magnetic fields are generated by stretching in large scale shear around the large-scale ring structure.

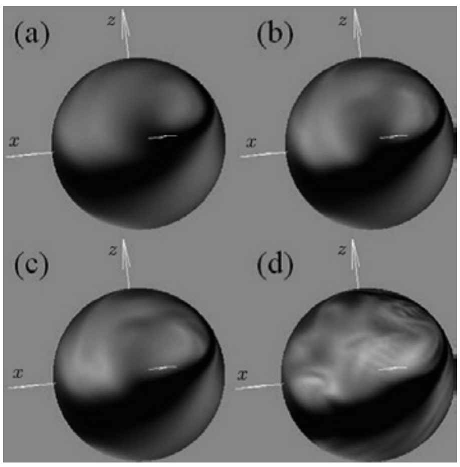


figure 2: Ring structure at different Reynolds numbers. Vorticity magnitude on the spherical surfaces of radius $r = 0.9$ are shown for (a) $Re = 1500$, (b) 1900, (c) 2000 and (d) 3000. Vorticity is larger in darker regions.[3]

[1] Kida, S. and Shimizu, S., *13th-ETC*, 268, (2011)
[2] Goto, S. et al., *Phys. Fluids*, **19**,061705 (2007)
[3] Koike, Y. et al., *JSST2012* (2012)