§9. Extended Gyrokinetic Field Theory for Time-dependent Magnetic Confinement Fields

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In conventional gyrokinetic studies, the gyrocenter phase-space variables are defined by using the background magnetic confinement field that is assumed to be independent of time. However, the background or equilibrium magnetic field changes along with the pressure profile on the transport time scale. Therefore, in order to accurately describe the long-time behaviors of the gyrokinetic turbulence, we need to treat the time-dependent background field and show how to determine its time dependence. In this work [1], the gyrokinetic field theory [2] is extended to derive the conditions which determine the time-dependent magnetic confinement fields in axisymmetric toroidal systems.

All governing equations for the gyrokinetic system considered here are derived from the variational principle, $\delta\mathcal{I} = 0$, where $\mathcal{I} \equiv \int_{t_1}^{t_2} L dt$ is the action integral and L is the Lagrangian which includes not only the gyrocenter coordinates and turbulent electromagnetic fields but also the constraint on the background magnetic fields. Then, besides equations for the gyrocenter motion equations and turbulent fields, the conditions which self-consistently determine the background fields varying on a transport time scale are obtained. The resultant gyrocenter motion equations are written in terms of the gyrocenter coordinates $\mathbf{Z}_a = (\mathbf{X}_a, U_a, \mu_a, \xi_a)$ for species a as

$$\frac{d\mathbf{X}_a}{dt} = \frac{1}{B^*_{a\parallel}}\left[\left(U_a + \frac{e_a}{m_a}\frac{\partial\Psi_a}{\partial U_a}\right)\mathbf{B}^*_a + c\mathbf{b}\times\left(\frac{\mu_a}{e_a}\nabla B_0 + \nabla\Psi_a + \frac{1}{c}\frac{\partial\mathbf{A}^*_a}{\partial t}\right)\right], \quad (1)$$

$$\frac{dU_a}{dt} = -\frac{\mathbf{B}^*_a}{m_a B^*_{a\parallel}}\cdot\left[\mu_a\nabla B_0 + e_a\left(\nabla\Psi_a + \frac{1}{c}\frac{\partial\mathbf{A}^*_a}{\partial t}\right)\right], \quad (2)$$

$$\frac{d\mu_a}{dt} = 0, \qquad \frac{d\xi_a}{dt} = \Omega_a + \frac{e_a^2}{m_a c}\frac{\partial\Psi_a}{\partial\mu_a}, \quad (3)$$

where the effects of the vector potential for the time-dependent background magnetic field appear through the terms proportional to $\partial\mathbf{A}^*_a/\partial t$ and the fluctuating electromagnetic fields are included in the potential Ψ_a. The gyrokinetic Vlasov equation for the gyrocenter distribution function $F_a(\mathbf{X}, U, \mu, t)$ is written as

$$\left(\frac{\partial}{\partial t} + \frac{d\mathbf{X}_a}{dt}\cdot\frac{\partial}{\partial\mathbf{X}} + \frac{dU_a}{dt}\cdot\frac{\partial}{\partial U}\right)F_a(\mathbf{X}, U, \mu, t) = 0. \quad (4)$$

where $d\mathbf{X}_a/dt$ and dU_a/dt represent the values of the right-hand sides of Eqs.(1) and (2) evaluated at the gyrocenter coordinates $(\mathbf{X}, U, \mu)$ and the time t. The gyrokinetic Poisson equation and the gyrokinetic Ampère's law are given by

$$\nabla^2\phi(\mathbf{x}, t) = -4\pi\sum_a e_a\int d^6\mathbf{Z}\, D_a(\mathbf{Z}, t)\delta^3(\mathbf{X} + \boldsymbol{\rho}_a - \mathbf{x}) \times\left[F_a(\mathbf{Z}, t) + \frac{e_a\widetilde{\psi}_a}{B_0}\frac{\partial F_a}{\partial\mu}\right], \quad (5)$$

and

$$\nabla^2(\mathbf{A}_0 + \mathbf{A}_1) = -\frac{4\pi}{c}(\mathbf{j}_G)_T, \quad (6)$$

respectively. The equilibrium magnetic field $\mathbf{B}_0$ is given in the axisymmetric form as $\mathbf{B}_0 = I\nabla\zeta + \nabla\zeta\times\nabla\chi$, where I and χ are determined from

$$I = \oint\frac{d\theta}{2\pi}\left[\frac{4\pi}{c}\overline{M_\zeta} + \overline{(B^{(\mathrm{gc})})_\zeta} - \overline{B_{1\zeta}}\right], \quad (7)$$

and

$$\Delta_*\chi = \overline{\left(\frac{4\pi}{c}\left[(\mathbf{j}^{(\mathrm{gc})})_T + \nabla\times\mathbf{M}\right] - \nabla\times\mathbf{B}_1\right)\cdot R^2\nabla\zeta} + \frac{\partial I}{\partial\chi}\overline{\Lambda_\zeta}, \quad (8)$$

respectively.

Equations (4)–(8) constitute the closed system of governing equations which determine F_a, ϕ, $\mathbf{A}_1$, I, and χ for the present gyrokinetic system with time-dependent axisymmetric background mangetic fields. Detailed definitions of all variables in the equations shown in this report are given in Ref.[1]. Conservation laws for energy and toroidal angular momentum of the whole system in the time-dependent background magnetic fields are naturally derived by applying Noether's theorem [1–3]. It is shown that the ensemble-averaged transport equations of particles, energy and toroidal momentum given in the present work agree with the results from the conventional recursive formulation with the WKB representation except that collisional effects are disregarded here [1].

1) H. Sugama, T.-H. Watanabe, M. Nunami, Phys. Plasmas **21**, 012515 (2014).
2) H. Sugama, Phys. Plasmas **7**, 466 (2000).
3) H. Sugama, T.-H. Watanabe, M. Nunami, Phys. Plasmas **20**, 024503 (2013).