§9. A Theory of Lead-Time in Probabilistic Excitation of L/H Transition

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A quantity of a lead-time, t_{lead}, is newly introduced to examine the probabilistic occurence of the L/H transition. The lead time is a time period during which a transition is likely occurs. We show that the lead-time has the statistical distribution as a function of the distance from critical parameter, e.g., $|n_c - n_{c0}|$ when the density is a key parameter for transition. It has the dependence like $t_{lead} \propto |n_c - n_{c0}|^2$ if the background noise distribution is given by the power law distribution with the index 2. (n_{c0} : critical density of transition in deterministic model.)

Transition phenomena in plasmas (e.g., L/H transition) are widely observed in various toroidal confinement devices. A transition (bifurcation) is modelled by the equation which includes a hysteresis and cusp-type bifurcation. There exist statistical variances in relevant variables. It is found that the transition occurs with a finite probability around the original threshold condition if the noise is present [1].

We here adopt two basic equations: One is the temporal evolution of the plasma parameter. The other is the dynamics of the loss rate that produces the hysteresis of the flux-gradient relation. We choose two representative variables, i.e., the density n and the loss rate γ in a layer with a finite width. The loss rate is directly related to the turbulence level and the particle diffusivity. The reduction to the 0-D model from the transport equation has been discussed [2]. The model equation used here takes the forms

$$\partial n / \partial t = S - \gamma n \qquad (1)$$

and

$$\zeta \partial \gamma / \partial t = n - 1 + a(\gamma - 1) - b(\gamma - 1)^3, \qquad (2)$$

where S is the particle influx into the layer, $\zeta (=O(B_p^2/B_t^2))$ is the dynamical time difference between n and γ, and the cubic equation $a(\gamma - 1) - b(\gamma - 1)^3$ describes the shape of the hysteresis. Appropriate normalization is used for n, γ and t.

We consider that parameters (S, a, b) are statistical variables and have fluctuation parts in time. We here mainly focus the effect of the statistical variances of a. We set $a = a_0 + \varepsilon_a$ and consider ε_s and ε_a as statistical variables. We study the probabilistic nature which is caused by the statistical property of the turbulence. Temporal evolution are investigated by solving eqs. (1) and (2). The variation in a hysteresis characteristic is examined. The variance of ε_a is taken account with $\bar{\varepsilon}_a = \sqrt{\langle a^2 \rangle - \langle a \rangle^2}$. The parameters are chosen as $a_0 = 0.5$, $b = 1.0$ and $\zeta = 0.01$. Accordingly, $n_{c0} \simeq 1.136$. We set $S = 1.0$. We here introduce the noise, the probabilistic function for ε_a which obeys the power law, i.e., $P(\varepsilon_a) \propto |\varepsilon_a|^{-2}$ in a certain region. (The index of 2 is chosen as an example, but this number is not far from the observation of nonlinear simulation.) In this example, the noise generator is used to obtain the fluctuation quantity ε_a in the domain $\varepsilon_a \geq 1 \times 10^{-6}$ and in $\varepsilon_a \leq -1 \times 10^{-6}$. In this region, the relation $P(\varepsilon_a) \propto |\varepsilon_a|^{-2}$ holds. We set $P(\varepsilon_a) = P(1 \times 10^{-6})$ $(=const.)$ for $|\varepsilon_a| \leq 1 \times 10^{-6}$. The variance is $\bar{\varepsilon}_a = 0.05$.

Oscillations like a limit cycle with irregular bursts are obtained. To show the probabilistic excitation of the plasma transition, the statistical distribution of the critical density n_c, at which the transition from H-mode to L-mode takes place, is observed. The finite probability of the transition below the criterion $n_c < n_{c0}$ occurs with the probability $P(n_c) \propto |n_c - n_{c0}|^{-2}$. This allows one to estimate the effective lead-time before the transition occurs.

We next study the transition from the H-mode to L-mode. A time sequence of repeated transitions (total number of I_{tot}) in dithers is considered as a set of I_{tot} observations of the transition events. In each temporal evolution of transition, the initial condition is chosen as the H-state, and the onset of H- to L-transition is observed. The ratio that the plasma state remains the H-mode (the transition does not occur yet) is set N as the mean value for the case of I_{tot} times of the repeated transitions. The quantity $N/|dN/dt|$ is an estimate of the 'lead-time': t_{lead}. In this case investigated here, the lead-time is shown to have the dependence such as

$$t_{lead} \propto |n_c - n_{c0}|^2$$

as is examined in fig. 1.

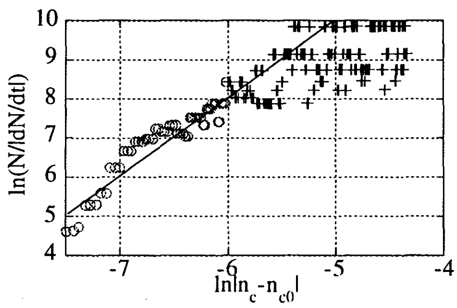

Fig.1 The dependence of $log_e(N/\dot{N})$ on $log_e|n_c - n_{c0}|$. The value of the slope for the solid line is 2.

References
[1] Toda, S. et al., 1998 ICPP & 25th EPS Conf. on Contr. Fusion and Plasma Physics, Praha, ECA Vol. **22C**, 1856 (1998).
[2] Itoh, S.-I. et al., Nucl. Fusion, **33**, 1445 (1993).