§1. A New Method Constructing Magnetic Flux Coordinate

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Magnetic coordinates are commonly used in the study of plasma in toroidal magnetic confinement devices. In constructing the magnetic flux coordinates, Boozer's algorithm[1] following the magnetic field line is widely used. In this paper the new method constructing the flux coordinates which is based on the algorithm to construct the generalized magnetic coordinates (GMC) is described.

The curvilinear coordinate system (ξ,η,ζ) is the generalized magnetic coordinates, ζ being the toroidal angle variable, when the magnetic field is expressed in the form

$$\boldsymbol{B} = \Phi(\xi, \eta)\nabla\xi \times \nabla\eta + \nabla\Psi(\xi, \eta, \zeta) \times \nabla\zeta. \quad (1)$$

If we introduce the magnetic flux densities

$$H^{\zeta} \equiv \sqrt{g}\boldsymbol{B}\cdot\nabla\zeta \equiv \boldsymbol{B}\cdot\frac{\partial \boldsymbol{r}}{\partial\xi}\times\frac{\partial \boldsymbol{r}}{\partial\eta},$$
$$H^{\xi} \equiv \sqrt{g}\boldsymbol{B}\cdot\nabla\xi \equiv \boldsymbol{B}\cdot\frac{\partial \boldsymbol{r}}{\partial\eta}\times\frac{\partial \boldsymbol{r}}{\partial\zeta}, \quad (2)$$
$$H^{\eta} \equiv \sqrt{g}\boldsymbol{B}\cdot\nabla\eta \equiv \boldsymbol{B}\cdot\frac{\partial \boldsymbol{r}}{\partial\zeta}\times\frac{\partial \boldsymbol{r}}{\partial\xi},$$

then, the expression (1) means that the H^{ζ} is required to be independent of toroidal angle ζ. If there exist the nested magnetic surfaces, the other two components H^{ξ} and H^{η} also become independent of ζ. The basic algorithm to construct the GMC is to deform the coordinates surfaces successively, so that the ζ-dependence part of flux densities decreases.

In the usual algorithm constructing GMC[2] the cylindrical coordinates is related to the coordinates as

$$r = R(\xi, \eta, \zeta) \equiv \xi + \sum_{n\neq 0} R_n(\xi, \eta)\exp(\mathrm{i}\, n\zeta),$$
$$z = Z(\xi, \eta, \zeta) \equiv \eta + \sum_{n\neq 0} Z_n(\xi, \eta)\exp(\mathrm{i}\, n\zeta), \quad (3)$$
$$\phi = \zeta.$$

The GMC is constructed by choosing the coefficients $R_n(\xi, \eta)$ and $Z_n(\xi, \eta)$.

If we consider the special case, such as the coordinate x is constant on the flux surface, then two conditions

$$H^{\xi} = 0, \quad \frac{\partial}{\partial\eta}H^{\eta} = 0 \quad (4)$$

are added to the original one

$$\frac{\partial}{\partial\zeta}H^{\zeta} = 0 \quad (5)$$

The relation between cylindrical coordinates and the magnetic coordinates is

$$r = R(\xi, \eta, \zeta) \equiv \sum_{m,n} R_{m,n}(\xi)\exp\{\mathrm{i}[m\eta + n\zeta]\},$$
$$z = Z(\xi, \eta, \zeta) \equiv \sum_{m,n} Z_{m,n}(\xi)\exp\{\mathrm{i}[m\eta + n\zeta]\}, \quad (6)$$
$$\phi = \zeta.$$

This time, the coordinate independent to ζ are also to be changed to satisfy conditions (4). The coordinate surfaces are modified to minimize the magnetic field component perpendicular to the ξ = constant surface.

Since the method does not assume the existence of the magnetic surface, the magnetic island exist can be detected as the resonant component of the H^{ξ} on the rational surface. On the other hand, the method gives the possibility to improve the numerical efficiency in construction of the GMC in the case of the good magnetic surfaces.

[1] A. Boozer: Phys. Fluids **25**, 520 (1982).

[2] M. Kurata, J. Todoroki: J.Plasma Fusion Res. SERIES, Vol.1 (1998) 491.