§14. Electrostatic Potential due to Induced Charge of Spherical Dust in Non-Uniform Electric Field

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The induced charge on a spherical dust immersed in non-uniform electric field is studied theoretically. The spherical conducting dust with radius R_d is attached on the infinitely extended conducting plane wall. The potential ϕ_{in} due to the induced charge satisfies the Laplace equation:

$$[\frac{\partial}{\partial\alpha}(\frac{\alpha}{\alpha^2+\beta^2}\frac{\partial}{\partial\alpha})+\frac{\partial}{\partial\beta}(\frac{\alpha}{\alpha^2+\beta^2}\frac{\partial}{\partial\beta})]\phi_{in}=0 \qquad (1)$$

in (α, β) space [1], where $\alpha = R_d r / (r^2 + z^2)$, $\beta = R_d z / (r^2 + z^2)$, and the origin of the cylindrical coordinates r and z is the contact point of the dust on the wall. The plane $z = 0$, i.e. $\beta = 0$, corresponds to the wall surface and $\beta = 1/2$ indicates the spherical dust surface, respectively. The external one-dimensional non-uniform electric field without the dust is approximated by the polynomial of the axial coordinate z: $\phi_{ex}(z)=\sum_{k=1} h_k z^k$. After taking into account the boundary conditions at the dust surface, we can obtain the local electrostatic potentials $\phi_{ex} + \phi_{in}$ in (α, β) space. The induced charge Q_{din} is calculated from the electric field to the normal direction E_n at the dust surface ($\beta = 1/2$):

$$Q_{din}=\int_{S_d}\sigma_s dS=\varepsilon_0\int_{S_d}E_n\big|_{\beta=1/2}dS$$

$$=-2\pi\varepsilon_0 R_d^2\int_{\alpha=0}^{\infty}\frac{\alpha}{(\alpha^2+\beta^2)^2}\frac{\partial}{\partial\beta}(\phi_{ex}+\phi_{in})\big|_{\beta=1/2}d\alpha\ , \qquad (2)$$

where σ_s and S_d are the surface charge density and the dust surface area, respectively. Finally the induced charge Q_{din} is obtained as a function of the dust radius R_d:

$$Q_{din}=-2\pi\varepsilon_0 R_d^2(c_1h_1+c_2h_2R_d+c_3h_3R_d^2+\cdots). \qquad (3)$$

Here the coefficients c_k's are the numerical constants, which are expressed by the Gamma and Rieman's Zeta functions. The first term of the RHS corresponds to the charge in the uniform electric field.

This result can be applied to the non-uniform potential in the Debye sheath. Our model of the Debye sheath formation is following: 1) Ions are monoenergetic, 2) Electrons have a truncated Maxwellian velocity distribution due to the absorption of high energy component by the wall, 3) The electric field is vanishing at the sheath entrance, and 4) The external electrostatic potential ϕ_{ex} is expressed by the polynomial of degree three, where the coefficients h_k's are determined by the potential drop across the Debye sheath and the sheath width. The induced charge Q_{din} is shown as a function of the dust radius R_d in Fig.1 for the relatively shallower potential drop $-e\phi_w/T_e = 3.5$ and deeper potential drop of 10.0, where the Debye sheath width is 8.0 λ_{Dse}, where λ_{Dse} is the electron Debye length at the sheath entrance.

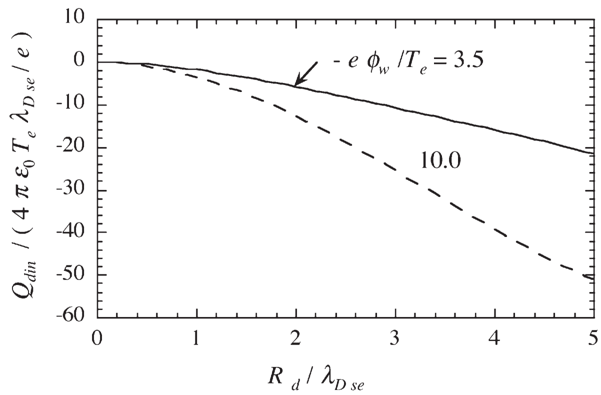

Fig.1 The induced dust charge as a function of the dust radius R_d for the cases of the potential drop $-e\phi_w/T_e = 3.5$ and 10.0, where the Debye sheath width is 8.0 λ_{Dse} .

The stronger electric field at the wall, which is made by the deeper wall potential, generates the higher induced charge. In case of the smaller dust than the Debye sheath width, the total charge approaches to the induced charge due to the non-uniform electric field, because the effect of plasma shielding is smaller than the effect of the induced electric field. These results can be compared to the results from the particle computer simulation [2], where the total charge includes both of effects, induced charge and plasma shielding. This theoretical analysis is important to understand the dust behavior in the boundary plasma [3].

References

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3) Krasheninnikov, S., Y. Tomita, Y., Smirnov, R., and Janev, R., Phys. Plasma, **11** (2004) 3141.