§10. Plasma-Sheath Formation in a Nonuniform Magnetic Field

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Condition for stable plasma-sheath formation in a nonuniform magnetic field is analytically obtained. Sheath and presheath formation near the symmetric axis in a nonuniform magnetic field has been studied analytically and numerically.1,2) The condition for stable sheath formation in a magnetic field has not been clarified in these studies. In this study an axisymmetric magnetic field with a monotonically decreasing axial profile near the axis is considered. A conducting wall, which is assumed to be perfectly absorbing and electrically floating, is installed vertically to the axis. As a width of sheath is as short as few Debye-lengthes, which is much shorter than mean-free-pathes of ionization and collisions, in the Debye sheath there is no plasma source and the change of magnetic field strength is quite small. In this collisonless sheath ion Boltzmann equation is described by the constants of motion of plasma ions are total energy ε and magnetic moment μ,

$$v_z(z,\varepsilon,\mu)\frac{\partial f(z,\varepsilon,\mu)}{\partial z}=0, \qquad (1)$$

where z is the axial length to the wall and axial velocity v_z should be expressed by z, ε and μ as follows.

$$v_z(z,\varepsilon,\mu)=\sqrt{2[\varepsilon-\mu B(z)-q\phi(z)]/M}. \qquad (2)$$

The quantities q, M, and ϕ denote the ion charge, mass, and electrostatic potential, which decreases monotonically to the wall, respectively. The Boltzmann equation of eq.(1) indicates ion distribution function in the sheath is expressed by that at the plasma-sheath boundary ($z = z_b$).

$$f(z,\varepsilon,\mu)=f(z_b,\varepsilon,\mu)\equiv f_b(\varepsilon,\mu). \qquad (3)$$

This distribution function gives the ion density in the sheath:

$$n_i(z)=\frac{2\pi B(z)}{M^2}\int_0^\infty d\mu \int_{q\phi_b+\mu B_b}^\infty d\varepsilon \times f_b(\varepsilon,\mu)/\sqrt{2[\varepsilon-\mu B(z)-q\phi(z)]/M} \qquad (4)$$

In order to study the behavior of electrostatic potential near the plasma-sheath boundary, the ion density is expanded near the plasma-sheath boundary.

$$n_i(z)=\frac{2\pi B_b}{M^2}(1-\frac{\Delta B}{B_b})\int_0^\infty d\mu \int_{q\phi_b+\mu B_b}^\infty d\varepsilon \times\frac{f_b(\varepsilon,\mu)}{\sqrt{2(\varepsilon-\mu B_b-q\phi_b)/M}} \times[1-\frac{q\Delta\phi+\mu\Delta B}{2(\varepsilon-\mu B_b-q\phi_b)}+\cdots]$$

$$\cong n_{ib}[1-\frac{q}{M}<v_z^{-2}>_b\Delta\phi-(1+\frac{1}{2}<v_\perp^2/v_z^2>_b)\frac{\Delta B}{B_b}], \qquad (5)$$

here $<\cdot\cdot>_b$ stands for the average to the ion distribution function at the plasma-sheath boundary. As the variation of ΔB is connected to that of $\Delta\phi$ by the relation:

$$\Delta B=-\left.\frac{dB/dz}{d\phi/dz}\right|_b\Delta\phi, \qquad (6)$$

the ion density near the boundary is expressed by the variation of potential $\Delta\phi$.

$$n_i(z)\cong n_{ib}\{1-[\frac{q}{M}<v_z^{-2}>_b -(1+\frac{1}{2}<v_\perp^2/v_z^2>_b)\left.\frac{d(\ln B)/dz}{d\phi/dz}\right|_b]\Delta\phi\} \qquad (7)$$

The electron density with the Boltzmann distribution may be expanded as follows:

$$n_e(z)=n_{eb}[1-e\Delta\phi/T_e+\cdots], \qquad (8)$$

here electron temperature is denoted by T_e. These expanded expressions of densities near the boundary determine the potential behavior through the Poisson's equation:

$$\varepsilon_0\frac{d^2\Delta\phi}{dz^2}\cong\frac{en_b}{T_e}[1-\frac{ZT_e}{M}<v_z^{-2}>_b -\frac{T_e}{e}(1+\frac{1}{2}<v_\perp^2/v_z^2>_b)\left.\frac{d(\ln B)/dz}{d\phi/dz}\right|_b]\Delta\phi. \qquad (9)$$

This equation gives the condition of the stable sheath formation in the Debye sheath:

$$<v_z^{-2}>_b^{-1}\geq\frac{ZT_e/M}{1-\alpha}, \qquad (10)$$

the parameter α, which indicate the effect of a nonuniform magnetic field, is defined:

$$\alpha\equiv\frac{T_e}{e}(1+\frac{1}{2}<v_\perp^2/v_z^2>_b)\left.\frac{d(\ln B)/dz}{d\phi/dz}\right|_b. \qquad (11)$$

In our case, where the strength of magnetic field is decreasing to the wall as well as electrostatic potential, the parameter α has a value of positive definite, which means the generalized Bohm criterion for the case of nonuniform magnetic field is restricted compared to that of the uniform magnetic field 3).

$$<v_z^{-2}>_b^{-1}\geq\frac{ZT_e/M}{1-\alpha}\geq\frac{ZT_e}{M}. \qquad (12)$$

The study of the case of increasing magnetic field strength and the two dimensional distribution of magnetic field lines are left for the future.

References

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