

§ 3. Theory of a Triple Vortex

Vranješ, J., Poedts, S. (K.U. Leuven),
Okamoto, A. (Nagoya Univ.),
Kono, M. (Chuo Univ.),
Yoshimura, S., Tanaka, M.Y.

Tripolar vortices are known from the experiments with rotating ordinary fluids as well as from the satellite observations in the nature. A stationary tripolar vortex has been observed in the Bay of Biscay with dimensions of about 70 kilometers. In plasma science travelling tripolar structures were predicted in various situations as local and global vortices. Typically they appear due to some spatial inhomogeneity of the equilibrium quantities like velocity, or density. Therefore it seems that they represent one possible natural form of self-organization in nonuniform fluid-like media. Recently, in a series of experiments with an argon plasma, stationary coherent structures in the form of global triple vortices were observed in a cylindrical plasma.

We propose here an analytical model, which takes into account the momentum input through the ion-neutral interactions, and construct non-linear solutions that could be able to describe the observed phenomena. We take a cylindrical plasma with the magnetic field oriented along its axis $\vec{B}_0 = B_0 \vec{e}_z$, and neglect the axial nonuniformity of the field and density, which is justified by the experimental fact that the characteristic scale length of inhomogeneity is much longer in the axial than in the radial direction. The plasma and neutral densities are r -dependent, and the system is subject to a r -dependent mainly poloidal rotation with the velocity $\vec{v}_0(r) = v_0(r) \vec{e}_\theta$. The equation of motion for ions is given by $m_i n_i (\partial/\partial t + \vec{v}_i \cdot \nabla) \vec{v}_i = q_i n_i (-\nabla \phi + \vec{v}_i \times \vec{B}_0) - \nabla p_i - m_i n_i \nu_i \vec{v}_i + \vec{F}_{ni}$, where ν_i is the frequency of ion-neutral collisions. The force \vec{F}_{ni} describes the momentum transferred in the ion-neutral charge transfer interactions which is given by $\vec{F}_{ni} = \nu_i m_i n_i \vec{v}_n$. The neutral flow \vec{v}_n is determined by the diffusion flux, $\vec{v}_n = -D_n \nabla n_n / n_n$, where D_n is the diffusion constant of neutrals, and n_n is the density of neutrals, so we have $\vec{F}_{ni} = -m_i n_i \nu_i D_n \nabla \log n_n$. The radially dependent distribution of neutrals affects the dynamics of plasma ions, resulting in an effective

pressure. Since the profile of neutrals in the experiment is usually concave, the effective pressure is directed inside, opposing the radial electric field and making an anti- $\vec{E} \times \vec{B}$ direction of rotation. From the ion continuity equation, with the help of quasineutrality condition we obtain the following equation for the potential:

$$\left[\frac{\partial}{\partial t} + \frac{1}{B_0} \vec{e}_z \times \nabla_\perp (\Phi_1 + B_0 \varphi) \cdot \nabla_\perp \right] \times \left[(1 - \rho^2 \nabla_\perp^2) \times (\Phi_1 + B_0 \varphi) - B_0 (\varphi + \psi) \right] + \frac{T_e}{e} \nabla_\parallel \cdot \vec{v}_\parallel = 0.$$

Here $\vec{v}_{i0} \equiv \vec{e}_\theta d\varphi(r)/dr = \vec{e}_\theta (d\Phi_0/dr)/B_0 - [1/(r\Omega_{ef} B_0^2)] (d\Phi_0/dr)^2 \vec{e}_\theta$, $v_* \equiv d\psi(r)/dr = -[(dn_0/dr) c_s^2] / n_0 \Omega_{ef}$, $\rho = c_s / \Omega_{ef}$, $c_s^2 = T_e / m_i$. We search for stationary states that results in coherent structures corresponding to the experiment. The solution for the electrostatic plasma potential (equilibrium plus perturbed) describing the tripolar vortex in terms of Bessel functions is given by $\phi_E(r, \theta) = -\phi_n(r) + \phi_n(r_0) + b(r^2 - r_0^2) + a_0 [J_0(kr) - J_0(kr_0)] + a_2 J_2(kr) \cos 2\theta$. Here ϕ_n is the effective potential due to the neutral pressure.

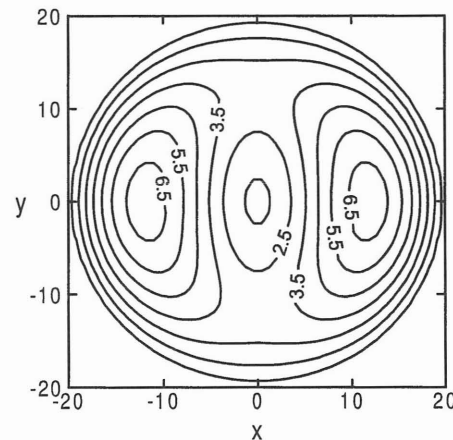


Fig.1 Contour plot of the solution

Reference

- J. Vranješ *et al.*, Phys. Rev. Lett. **89** (2002) 265002
A. Okamoto *et al.*, Phys. Plasmas **10** (2003) 2211.