§16. An Improved δf Monte Carlo Simulation for Collisional Transport

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A δf method, solving drift kinetic equation, for collisional transport calculation is presented. A new scheme is considered to evaluate the marker density g in weight calculation. The present scheme employs an additional weight function to directly solve g from its kinetic equation. Therefore the severe constraint that the real marker distribution must be consistent with the initially assumed g is relaxed. An improved like-particle collision scheme is also presented [1]. By performing compensation for momentum, energy and particle losses, the conservations of all the three quantities are greatly improved during collisions.

Setting $f = f_0 + f_1$ $(f_0 \gg f_1)$, we rewrite the drift kinetic equation for f_1 as

$$\frac{Df_1}{Dt} = -\vec{v_d} \cdot \nabla f_0 + C(f_0, f_1) \tag{1}$$

$$\frac{Df}{Dt} = \frac{\partial f}{\partial t} + (\vec{v_\parallel} + \vec{v_d}) \cdot \nabla f - C(f, f_0) \tag{2}$$

The simulation particles are described by a marker distribution function, $F_M(\vec{x}, \vec{v}, w, t)$, in the extended phase space $(\vec{x}, \vec{v}, w)$, which satisfies

$$\frac{D}{Dt}F_M + \frac{\partial}{\partial w}(\dot{w}F_M) = S_M(\vec{x}, \vec{v}, w, t), \tag{3}$$

where S_M denotes a particle source. Set that f_1 and F_M have the following relation

$$f_1(\vec{x}, \vec{v}, t) = \int wF_M dw. \tag{4}$$

The weight equation is determined by requiring that equations (1), (2), (3), and (4) are consistent with each other. The weight equation is determined as

$$\dot{w} = \frac{1}{g}\left[-\int wS_M dw - \vec{v_d} \cdot \nabla f_0 + C(f_0, f_1)\right] \tag{5}$$

$$g(\vec{x}, \vec{v}, t) \equiv \int F_M(\vec{x}, \vec{v}, w, t) dw \tag{6}$$

$$\frac{D}{Dt}g = \int S_M dw. \tag{7}$$

We now consider to solve g from equation

$$\frac{D}{Dt}G_M + \frac{\partial}{\partial \omega}(\dot{\omega}G_M) = \Omega_M(\vec{x}, \vec{v}, \omega, t) \tag{8}$$

$$g_1(\vec{x}, \vec{v}, t) = \int \omega G_M d\omega. \tag{9}$$

By the same way, we obtain the weight ω as

$$\dot{\omega} = \frac{1}{h}\left[-\int \omega\Omega_M d\omega - \vec{v_d} \cdot \nabla f_0 + \int S_M dw\right], \tag{10}$$

$$h \equiv \int G_M(\vec{x}, \vec{v}, \omega, t) d\omega, \tag{11}$$

$$\frac{D}{Dt}h = \int \Omega_M d\omega. \tag{12}$$

We, next, solve h by the δf method and we will have a hierarchy for weighting equations. To trancate the hierarchy, we employ the nonlinear weighting method for h.

With the improvement in both like-particle collision scheme and weighting scheme, the δf simulation shows a significantly upgraded performance. Ion neoclassical transport with finite orbit width dynamics is calculated over entire poloidal cross section. Ion thermal transport near magnetic axis shows a great reduction from its conventional neoclassical level, like that of previous δf simulation. On the other hand, the direct particle loss from confinement region may strongly increase ion energy transport near the edge. It is found that ion parallel flow near the axis is also largely reduced due to non-standard orbit topology.

References

[1] W.Wang et al., NIFS-588, March 1999.