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THREE-DIMENSIONAL PARTICLE SIMULATION STUDY ON STABILIZATION OF THE FRC TILTING INSTABILITY

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key words: field reversal configuration, tilting instability, stabilization, particle effect, particle simulation, meandering particle

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THREE-DIMENSIONAL PARTICLE SIMULATION STUDY ON STABILIZATION OF THE FRC TILTING INSTABILITY

Abstract

By carrying out a three-dimensional nonlinear particle simulation in the cylindrical coordinates it is disclosed that ion kinetic effects act to stabilize the FRC tilting instability and to form an anisotropic temperature distribution. For the case of $\bar{s} \approx 1$ a large number of ions execute a large amplitude oscillatory motion around the field-null line and carry most of the ion toroidal current, where $\bar{s}$ measures the number of ion gyroradii over the radial distance between the magnetic separatrix line and the field-null line. It is found that this motion, which is called a meandering motion, plays an important role in keeping the FRC plasma stable against tilt disruption and in forming the temperature anisotropy.

1. Introduction

The magnetohydrodynamic(MHD) linear theory[1] predicts that the field-reversed configuration (FRC) plasma is unstable against the tilting instability, while no experimental evidence has so far been reported on the tilt disruption. Two possibilities have been considered to explain the discrepancy between the MHD linear theory and the experiment. The first explanation is that the nonlinear saturation mechanism could protect the FRC plasma from the destructive growth of the tilt mode. In this respect Horiuchi and Sato's [2] three-dimensional full MHD simulation found no evidence for the nonlinear saturation of the tilt mode except for a highly spinning case.

An alternative explanation is that the stabilization effect due to the ion finite-Larmor radius (FLR) can operate effectively in the currently operating devices since the plasma confinement scale is comparable to the ion Larmor radius. Barnes et al.[3] derived the linear growth rate from the Vlasov-fluid dispersion equations and found that the tilt mode could be stabilized for a large gyroradius case of $\bar{s} \leq 2$, where $\bar{s}$ is defined by

$$\bar{s} = \int_{r_a}^{r_s} \frac{rdr}{r_s \lambda_i}, \tag{1}$$

r_s is the separatrix radius, r_a is the radius of the magnetic null, and λ_i is the local ion gyroradius. By solving the MHD equations with the Hall term Ishida el al.[4] and Milroy et al.[5] have shown that the Hall term could reduce the growth rate of the tilt mode for the highly prolate and small $\bar{s}$ case. However, they could not satisfactorily explain the discrepancy between the theory and the experiment. This is because the Hall term can represent only a part of the ion FLR effect.

In order to investigate fully the FLR stabilization effect against the tilting instability we carry out a macro-scale particle simulation that can describe both the electron and ion FLR effects and the global behavior over the device scale simultaneously[6].

2. Simulation model

We study the FRC plasma in a cylindrical conducting vessel in which plasma is confined by a uniform external field. The equations to be solved are the equations of motion

$$\frac{d(\gamma_j \mathbf{v}_j)}{dt} = \frac{q_j}{m_j} [\mathbf{E} + \frac{\mathbf{v}_j}{c} \times \mathbf{B}], \tag{2}$$

$$\frac{d\mathbf{x}_j}{dt} = \mathbf{v}_j, \tag{3}$$

and the Maxwell equations

$$\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E}, \tag{4}$$

$$\frac{1}{c}\frac{\partial \mathbf{E}}{\partial t} = -\nabla \times \mathbf{B} - 4\pi \mathbf{j}, \tag{5}$$

$$\nabla \cdot \mathbf{B} = 0, \tag{6}$$

$$\nabla \cdot \mathbf{E} = 4\pi\rho, \tag{7}$$

where $\mathbf{x}_j(t)$, $\mathbf{v}_j(t)$, m_j, q_j, γ_j, $\mathbf{j}(\mathbf{x},t)$ and $\rho(\mathbf{x},t)$ are the position, the velocity, the rest mass, the charge, the relativistic γ-factor of the j-th particle, the current density and the charge density, respectively. We solve the equations (2)-(5) in the cylindrical coordinates (r, ϕ, z) by assigning the initial conditions $\mathbf{x}_j(0)$, $\mathbf{v}_j(0)$, $\mathbf{B}(\mathbf{x}, 0)$ and $\mathbf{E}(\mathbf{x}, 0)$ which satisfy a two-fluid MHD equilibrium. The boundary condition is such that the physical quantities are periodic at two axial edges of the cylindrical vessel and a particle is completely elastically reflected on the conducting wall. Numerical scheme used for the three-dimensional particle simulation relies on a semi-implicit method[6]. Four simulation runs with different values of $\bar{s}$ are carried out by using a hundred thousand particles. The simulation runs are terminated after one Alfven transit time t_A where t_A is defined by r_0/v_A ; r_0 and v_A are the device radius and the average Alfven velocity in the plasma region.

3. Results

One of the characteristic features of the FRC plasma is that a field-null line exists in the central plasma region due to the strong toroidal plasma current. Figure 1 shows the initial profiles of the poloidal magnetic flux, the ion thermal pressure and the toroidal current density in the poloidal plane for $\bar{s} = 1$. It is worthy to notice in Fig. 1 that the FRC plasma is distributed in a fairly prolate region around the field-null line. Figure 2 shows the top view of the orbits of one hundred ions for the cases of $\bar{s} = 1$ (top) and $\bar{s} = 5$ (bottom) where each curve represents the projection of the ion trajectory onto the midplane (z = constant) during one Alfven transit time from the start of simulation. Most of ions in the vicinity of field-null line cannot make gyration motions but execute a meandering motion around the filed-null line without any self-intersections of orbits. The number of meandering ions increases and the oscillation amplitude of meandering motion becomes larger as $\bar{s}$ decreases.

Figure 3 shows the profiles of the electron distribution (top) and the ion distribution (bottom) in the (v_ϕ, v_z)-plane (right) and in the (v_r, v_z)-plane (left) for the case of $\bar{s} = 2$. The meandering ions oscillate with larger amplitude along the z-direction compared with that along the r-direction because the scale height of magnetic field strength in the z-direction is larger than that in the r-direction. The anisotropy of the meandering motion results in an anisotropic ion temperature, i.e., $T_z > T_r, T_\phi$ (bottom panel of Fig. 3). On the other hand, the electron distribution is almost isotropic because the number of meandering electrons is very small (top panel of Fig. 3).

The dependence of the average growth rate of the tilt mode on the parameter $\bar{s}$ is plotted in Fig. 4, where the open circle represents the value obtained by the simulation, and the filled triangles show the results of a linear theory[3]. The evolution of the tilt mode is completely suppressed when $\bar{s} \approx 1$. As $\bar{s}$ increases, the tilt mode tends to be more unstable and the growth rate approaches the MHD value. The behavior of the kinetic growth rate is in good agreement with the result of a linear theory. It can be concluded therefore that the stabilization effect due to the finiteness of the ion Larmor radius is very efficient for the FRC tilt mode.

4. Conclusion

Here we give a theoretical model to explain the FRC tilt stabilization in connection with the characteristic of a meandering motion. For the kinetic plasma of $\bar{s} = 1$ most of the ions are free from the constraint of the magnetic field and oscillate around the field-null point with a large amplitude. Suppose that a perturbation of $n = 1$ tilt mode is added to the velocity field of the meandering ions in a two-dimensional(axially symmetric) equilibrium. The ion changes to a new oscillation orbit the amplitude of which varies dependent on the phase difference between the meandering oscillation and the perturbation. However, the oscillation center of a new orbit remains the same. When the orbit is averaged over one oscillation period, therefore, the $n = 1$ tilt perturbation does not appear in the toroidal current carried by the meandering ions on the average. In other words, ions with meandering orbits do not contribute to the growth of the perturbation of the $n = 1$ tilt mode. We thus conclude that the ions with meandering orbits play a key role in keeping the system stable against the tilting perturbation, and that the evolution of tilt mode can be completely suppressed when most of the ions move on the stable meandering orbits, i.e., when $\bar{s} \approx 1$.

Acknowledgments

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Figure captions

Fig. 1. Initial profiles of the poloidal magnetic flux, the ion thermal pressure and the toroidal current density in the poloidal plane for $\bar{s} = 1$.

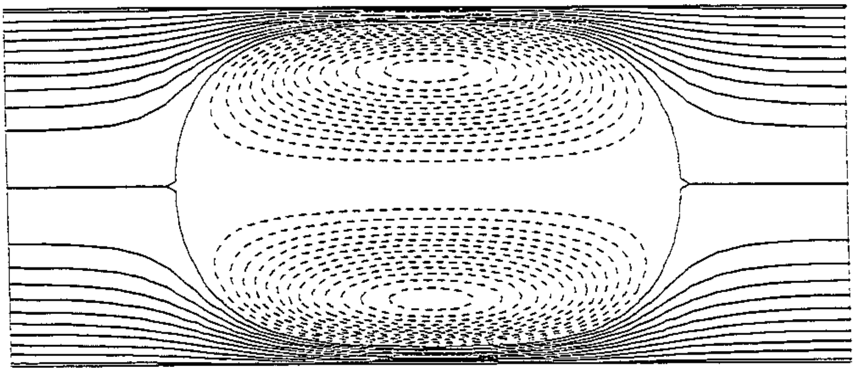
Fig. 2. Top views of the orbits of one hundred ions for the cases of $\bar{s} = 1$ (top) and $\bar{s} = 5$ (bottom).

Fig. 3. The electron distribution (top) and the ion distribution (bottom) in the (v_ϕ, v_z)-plane (right) and in the (v_r, v_z)-plane for the case of $\bar{s} = 2$.

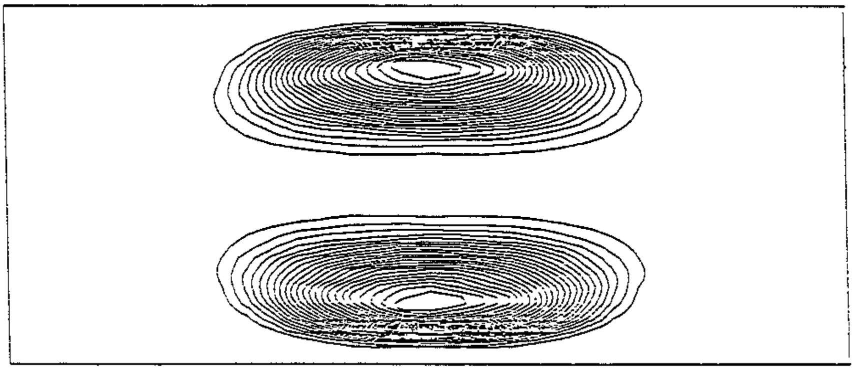
Fig. 4. The $\bar{s}$-dependence of the average growth rate of the tilting instability. The open circle represents the value obtained by the simulation, and the filled triangles show the results of a linear theory [Ref. 3]. The dashed line represents the average tendency of the growth rate as a function $1 / \bar{s}$.

Fig. 1

Poloidal Magnetic Flux



Ion Thermal Pressure



Current Density

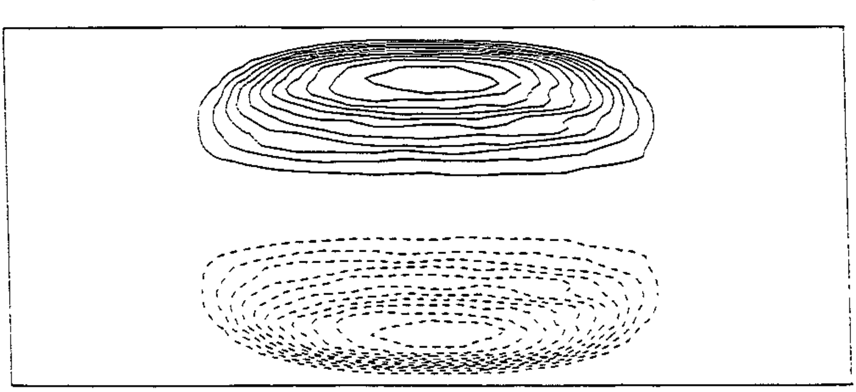
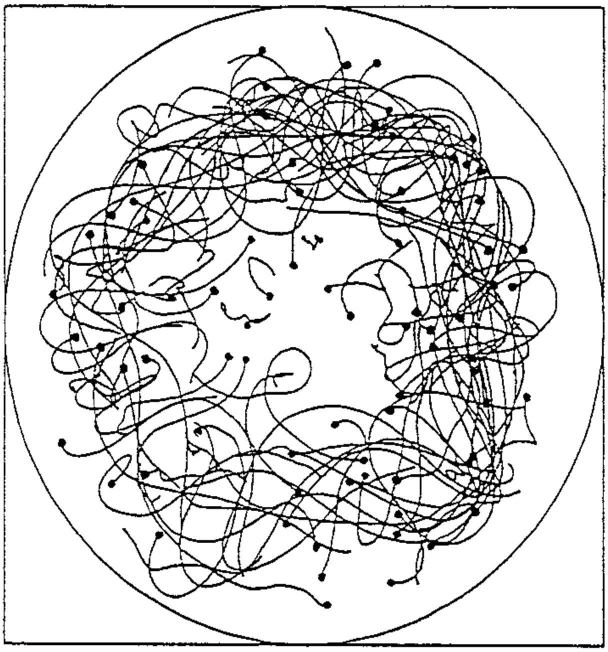


Fig. 2

$\bar{S}=1$



$\bar{S}=5$

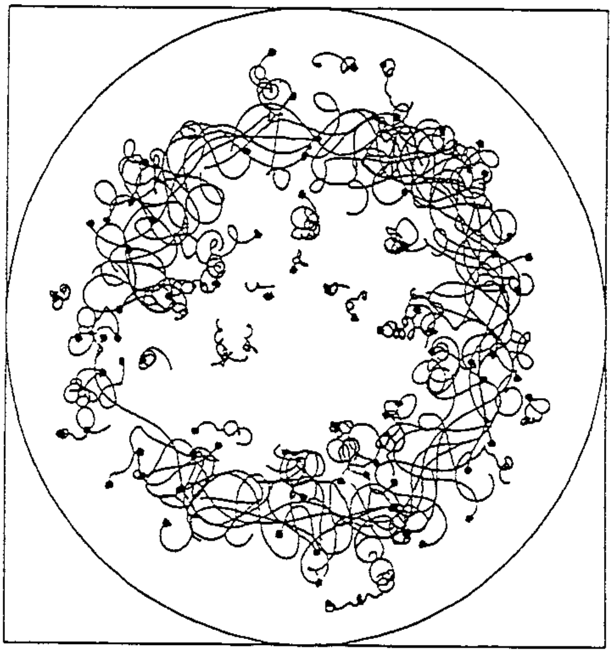
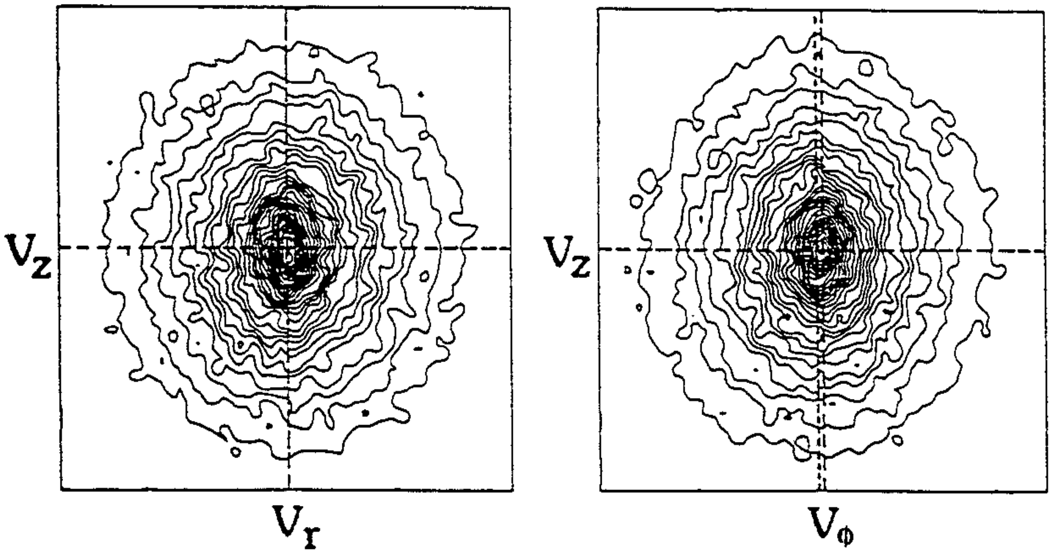


Fig. 3

Electron Distribution ($\bar{s}=2$)



Ion Distribution ($\bar{s}=2$)

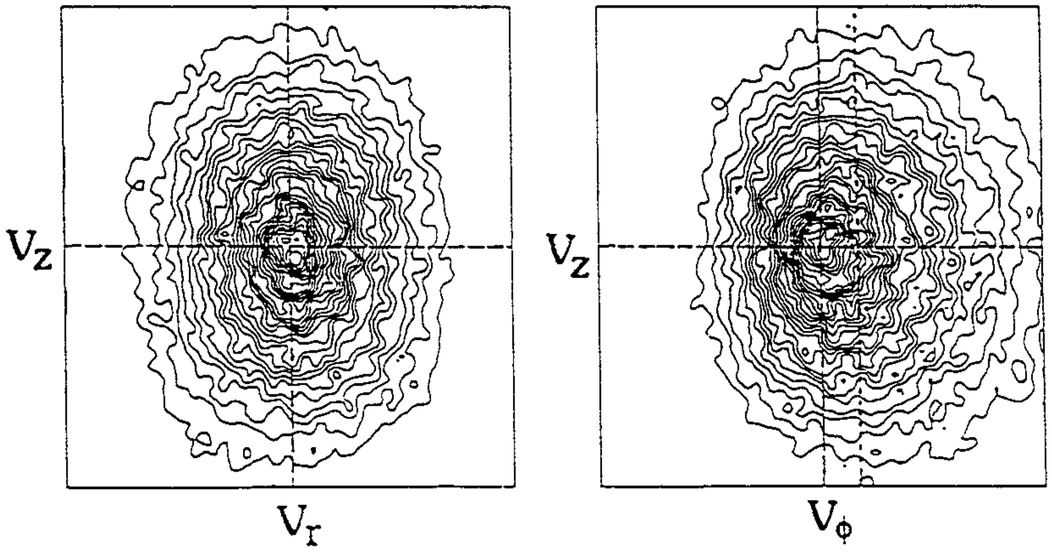
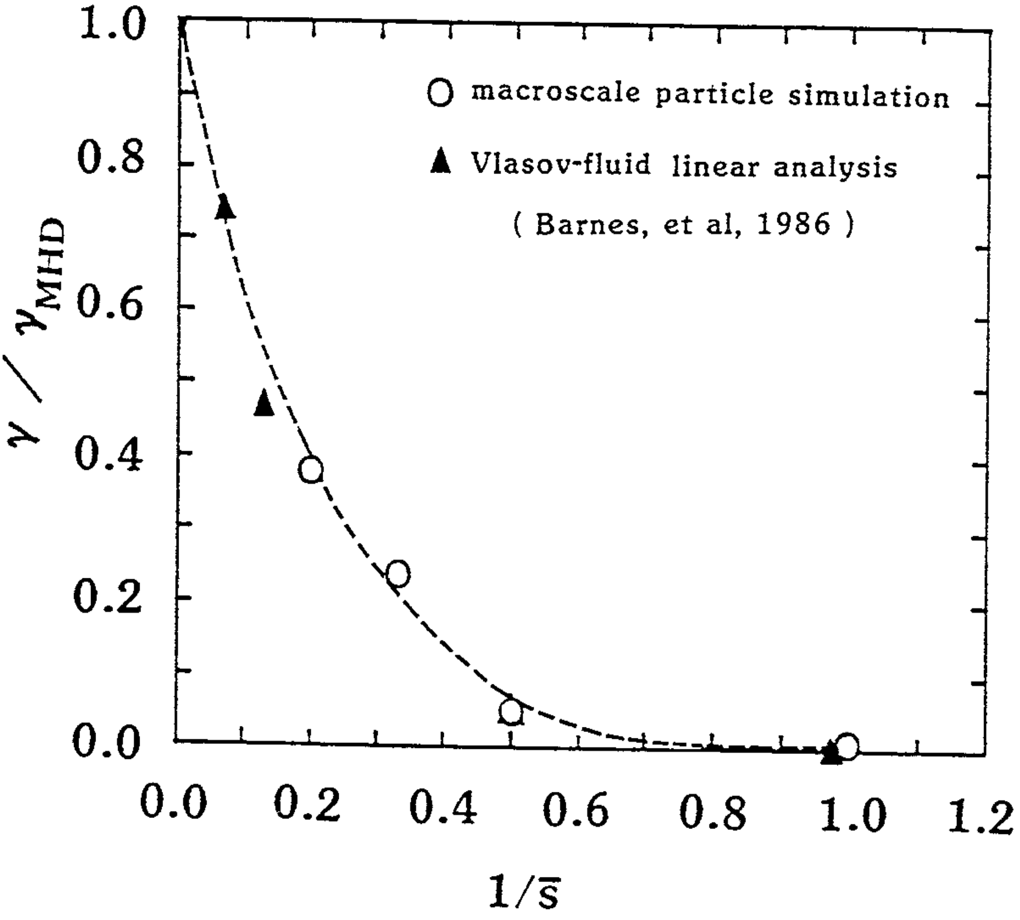


Fig. 4


○ macroscale particle simulation
▲ Vlasov-fluid linear analysis
(Barnes, et al, 1986)
γ / γ_{MHD}
1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0
1.2
$1/\overline{s}$