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Steady State Tokamak
Sustained by Bootstrap Current without Seed Current

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Abstract

Steady state tokamak solution is studied by taking into account the current diffusivity. The seed current for generating the Bootstrap current is supplied through the radial diffusion of the Bootstrap current. Steady state tokamak, without external current drive power, is in principle possible.

The diffusion-driven toroidal current[1-3], which has been known by the name of "Bootstrap current", has recently attracted attentions. The neoclassical theory of the plasma transport in toroidal plasmas has shown that the radial diffusion of plasma can drive the toroidal current. This mechanism would reduce the necessary circulating power for the future tokamak reactors. Developments of the theory were made in many aspects, and comparison with experiments were also enthusiastically investigated [4-9]. The mechanism to drive the Bootstrap current, however, cannot completely annihilate the current drive power. The Bootstrap current requires the "seed current" near the axis[1]. The necessity of the seed current is one of the candidates to limit the lower boundary of the circulating power to sustain the steady state tokamaks.

Since the Bootstrap current is driven by plasma diffusion, the effects of the many transport processes on the toroidal current must be carefully investigated. The anomalous transport mechanism to drive the current has been investigated[10,11]. It was found that the current drive by the anomalous transport, which would be responsible to the presently-observed anomalous loss of the plasma energy, remains to be a small correction. In this article we study the effect of the current diffusion across the magnetic surface. We find that the seed current near axis can be supplied by the radial diffusion of the Bootstrap current, which is driven at the location where the density has a large gradient. By supplying the seed current through current diffusion, the Bootstrap-current tokamak solution can exist stationarily without the external current drive scheme. This may give a possibility of the steady-state tokamak ignition.

We take a simple model of high aspect-ratio tokamak with circular cross section, of which the temperature is constant in space. (This does not limit the generality of the argument.) In the banana regime, the diffusion-driven current is given to the leading order of $\sqrt{\varepsilon}$ as[12],

$$J_{bs} = -4.88\sqrt{\varepsilon}\frac{p_e}{B_p}\frac{1}{n}\frac{dn}{dr} \tag{1}$$

where $\varepsilon=r/R$, r is the minor radius, R is the major radius, J_t is the toroidal current, B_p is the poloidal magnetic field, p_e is the electron pressure and n is the electron density.

The necessity of the seed current to generate the Bootstrap current has been discussed[1]. This fact has been thought to set a lower limit for the external current drive. The diffusion of the current to the plasma axis can supply the seed current, which is amplified by the Bootstrap current.

We study the steady state with zero toroidal electric field. In the presence of the current diffusivity[13,14] the term which is written as $\lambda\nabla^2 J_t$ is added to Ohm's law. We have

$$\eta(J_t - J_{bs}) - \lambda\nabla^2 J_t = 0, \tag{2}$$

for the stationary state. Equation (2) with Ampere's law, rB_p =

$\int_0^r \mu_0 J_t r dr$, can be written as

$$\lambda \frac{1}{s}\frac{d}{ds} s \frac{d}{ds} J_t - \eta J_t + C\, s^{3/2} \frac{I(1)^2}{I(s)} = 0 \qquad (3)$$

where

$$I(s) = \int_0^s J_t(s) s ds, \qquad (4)$$

$$C = 4.88 q(a)^2 \nu \rho^2 \left(\frac{R}{a}\right)^{3/2} \mu_0 \quad \frac{1}{sn}\frac{dn}{ds}, \qquad (5)$$

s is the normalized radius s=r/a (a is the minor radius of the tokamak), q(a) is the safety factor at the surface, η is the classical resistivity, $m_e \nu / ne^2$, ν is the electron collision frequency and ρ is the electron gyroradius. Since the density profile is flat at the axis, C reduces to a constant near axis. Equation (3) allows, in the case of $\lambda=0$, only the trivial solution $J_t = 0$, if the condition that J_t and n(s) are regular at the axis (s=0) is required. (This result corresponds to the necessity of the seed current.) If λ does not vanish, Eq.(3) allows the nontrivial solution with the requirement of the boundary condition

$$\frac{d}{ds} J_t = 0 \quad \text{(at } s=0\text{)}. \qquad (6)$$

In the vicinity of the axis, s=0, the first and second terms in Eq.(6) dominates. The solution of Eq.(3) is approximately given as

$$J_t(s) \simeq J_0 I_0(a\sqrt{\eta/\lambda}s), \tag{7}$$

where I_0 is the 0th order Bessel function of the second kind and J_0 is the current density at the axis. The fact of dJ/ds>0 indicates that the current is diffusing to the axis.

The energy balance is studied. By multiplying J(s)s to Eq.(3) and integrating it from zero to 1, we have the energy balance relation per unit toroidal length as

$$\lambda J_t(1)J_t' = \int_0^1 \lambda |J_t'|^2 sds + \int_0^1 \eta J_t^2 sds - \int_0^1 C\frac{I(1)^2}{I(s)} Js^{5/2}ds \tag{8}$$

The left hand side is the power supplied across the plasma surface. The first term in the right hand side is the dissipated power by the current diffusion, the second is the Ohmic dissipation, and the third is the work done by the diffusion-driven electro-motive force. The parameter C is proportional to the plasma beta value. The first and second terms in the right hand side are positive definite, and the third one is negative. If the beta value is large enough, the right hand side of Eq.(8) goes to zero and the supplied power can vanish.

The solution of Eq.(3) is characterized by two parameters,

$C(s)$ and λ/η. Numerical solution is obtained for the case of

$$C = C_0(1-s^2) \tag{9}$$

and C_0 is constant. Parameter λ/η is also taken to be constant in space for the simplicity. The parameter C_0 can be rewritten as

$$C_0 = \eta 4.88\sqrt{a/R}\beta_{ep} \tag{10}$$

where β_{ep} is the poloidal electron beta value $\{q(a)R/a\}^2\mu_0 p_e(0)/B^2$. Figure 1 illustrates the externally supplied power (left hand side of Eq.(8)) as a function of the electron beta value. As the plasma pressure increases, the externally supplied power starts to decrease; it finally vanishes at the critical beta value ($\sqrt{a/R}\beta_{ep}$ = 1.48 for this case). Above this critical beta value, the supplied power is negative, which implies that the negative toroidal electric field is generated for the constant current condition (recharging). Figure 2 shows the radial profile of the toroidal current. Solid line indicates the self-sustaining state, i.e., the stationary state is sustained without external current drive (slightly excess current is generated, as a whole, and is ejected from the plasma.) The diffusion-driven current, which is strongly generated near the half radius, diffuses inward, supplying the seed current at the axis. The toroidal current also diffuses outward.

In summary, we study the influence of the radial diffusion of the plasma current on the steady state solution of the tokamak. The toroidal current at the axis is generated by the current diffusion, and works as the seed current for the Bootstrap current. It is shown that the Bootstrap current can sustain itself by this mechanism. The steady state tokamak solution exists without being supplied the seed current externally. This implies the possibility of the steady state ignition in tokamaks.

This result indicates that the small amount of the current diffusivity is effective in reducing the necessary circulating power to sustain the toroidal current in high beta tokamaks. The practical application of this finding to the real experimental program is beyond the scope of this study. For instance, the stability analysis is inevitable since the plasma is in the very high beta regime. Similar research has recently been reported, in which large current diffusivity is generated by the global tearing mode[15]. We would like to emphasize that the further study is necessary to fully understand the diffusion-driven current in the high beta tokamaks.

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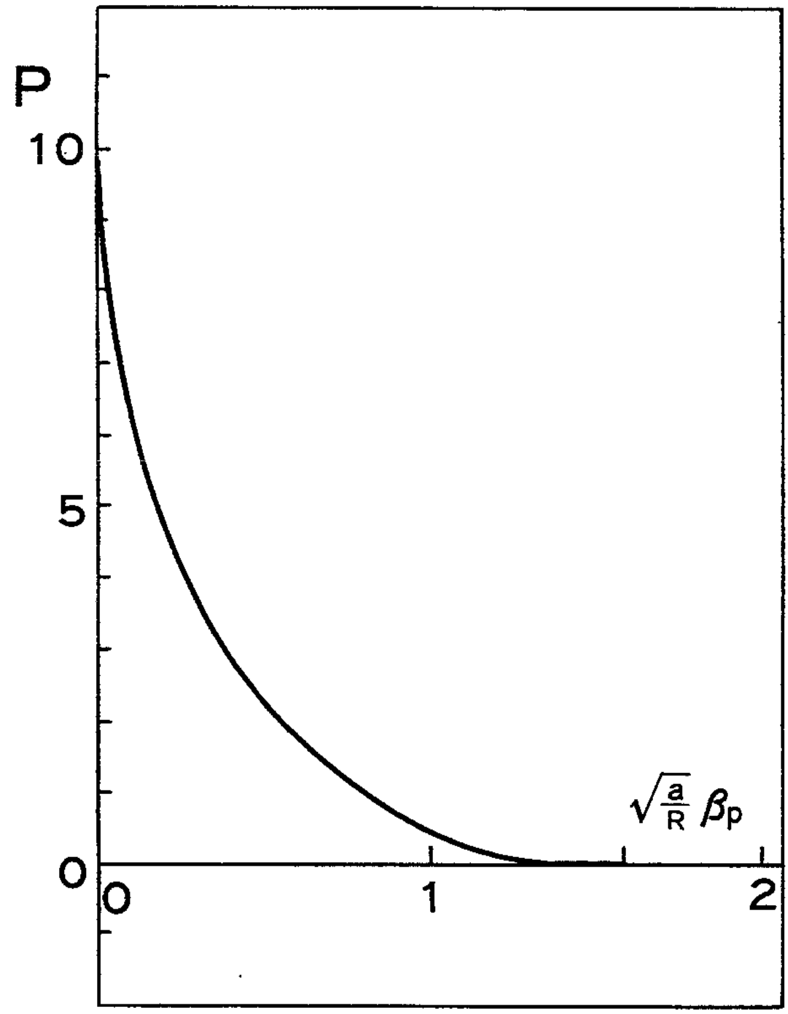
Figure Captions

Fig.1 Externally injected power as a function of $\sqrt{a/R}\beta_{ep}$ for the condition of fixed total current I_p. Unit of the input power is taken as $\eta I_p{}^2/2\pi a^2$. Parameter $a^2\eta/\lambda$ is chosen to be 100.

Fig.2 Radial profile of the toroidal current. Parameter $\sqrt{a/R}\beta_{ep}$ is chosen to be 1.58 (solid line), 0.497 (dashed line) and 0.170 (dashed-dotted line). Other parameter is the same as in Fig.1

Fig. 1



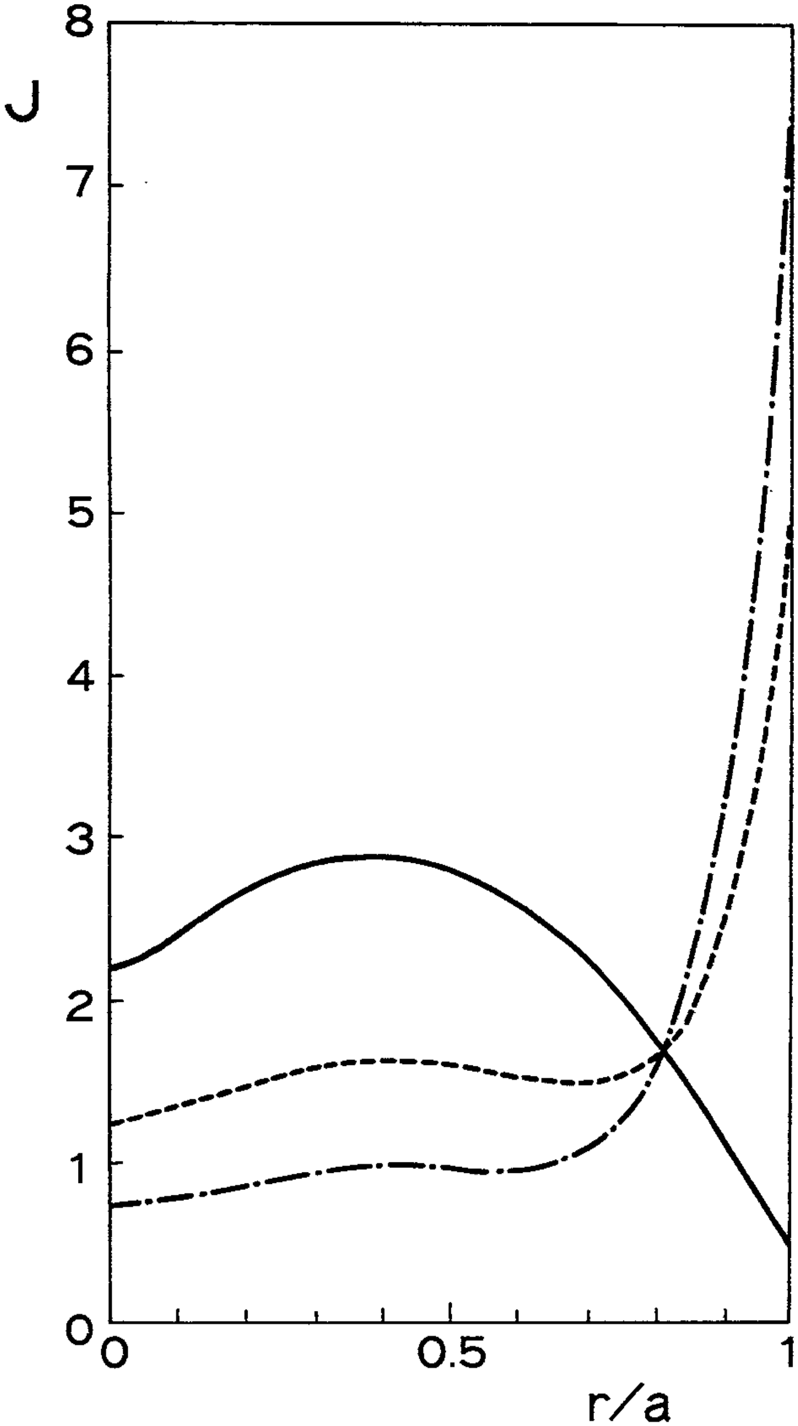


Fig. 2

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