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Comment on 'A Mean Field Ohm's Law for Collisionless Plasmas'

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Abstract

Recently, kinetic calculation of the current diffusivity (λ) was made and it was commented that the fluid model of anomalous transport, in which the self-sustained turbulence and L-mode transport has been obtained [Itoh et al., Phys. Rev. Lett. **69** (1992) 1050], has overestimated λ [Biglari, et al., Phys. Fluids B5]. This comment was misled by the improper evaluation of the wave number. The kinetic estimate of λ is in the same order of the one in the fluid model. This would be one of the reasons that the transport theory, which was derived by using the fluid equations, explains well the present experimental results.

Keywords: Current diffusivity, Kinetic theory, L-mode, Ohm's law

Recently a theory on the anomalous transport in toroidal plasmas has been developed[1,2]. The nonlinear destabilization through the diffusion of the current was found to be the key for the self-sustained turbulence. The solution of the renormalized mode equation was obtained[1-4], and the associated transport coefficient was found to be $\chi = (\lambda/\chi)\sqrt{\chi/\mu}\,\alpha^{3/2} f(s)^{-1} a^2/\tau_{Ap}$, where α is the parameter to denote the pressure gradient ($\alpha = q^2\beta'/\varepsilon$, ε is the inverse aspect ratio, β is the ratio of the plasma pressure to the magnetic pressure, $\beta' = d\beta/d(r/a)$, q is the safety factor), μ is the ion viscosity, λ is the current diffusivity, and χ is the thermal conductivity, f(s) denotes the effect of shear, a and R are plasma minor and major radius, respectively, and τ_{Ap} is the poloidal Alfvén transit time. This result shows the important role of the current diffusivity, and the ratio λ/χ is the crucial parameter. By renormalizing the reduced set of MHD equations, the estimate $\lambda/\chi \simeq (\delta/a)^2$ was obtained (δ is the collisionless skin depth. See Ref.[4] for detailed derivation.) This result was also confirmed by Connor by using the scale invariance method[5]. The result was applied to tokamaks, and a good agreement was found between the theoretical prediction and experimental result[4]. The ratio λ/χ was obtained for the collisionless plasma which is subject to the magnetic braiding, and a similar value was obtained, although the ratio μ/χ has shown a slight difference from the electrostatic estimate[6]. Since the theory in [1-4] was developed on the basis of the fluid model, the kinetic theory for the collisionless plasma is necessary in order to fully explain the experimental results on tokamaks.

The kinetic estimate of the cross-field transport of the electron current was obtained in Ref.[7], but the ratio λ/χ was not discussed in detail. Recently, Bigrali and Diamond has reported an analysis on the current diffusivity by performing the quasi-linear analysis on the drift kinetic equation[8]. By introducing the real frequency of the mode, ω_k, it was found that

$$D_J(\text{kinetic})/D_J(\text{fluid}) \simeq (\omega_k/\omega_{te})^3 \qquad (1)$$

where D_J is the diffusivity of the current in the mean field Ohm's law and ω_{te} is the electron transit frequency $k_\parallel v_{Te}$. D(kinetic) is the result of the quasi-linear theory on the collisionless limit and D(fluid) is our renormalized calculation. By taking the estimate $\omega_k \simeq \omega_{*e}$ (electron drift frequency, $k_\theta \rho_e v_{Te}/L_n$; v_{Te} is the electron thermal velocity, L_n is the density gradient scale length, ρ_e is the electron gyroradius, and k_θ is the poloidal wave number) and $k_\parallel \sim 1/R$, i.e.,

$$\omega_k/\omega_{te} \sim (R/L_n)k_\theta \rho_e, \qquad (2)$$

it was claimed that $\omega_k/\omega_{te} \ll 1$ and that our results overestimated the current diffusivity.

The claim in Ref.[8], that we have overestimated the current diffusivity, however, was misled by a careless estimation of the ratio ω_k/ω_{te}. No plausibility argument was made for the estimates of k_θ and $k_\parallel$ in [8]. The typical wave number of the mode, which is destabilized by the nonlinear interactions, was obtained as[1-4)]

$$k_\theta = h(s)\delta^{-1}\alpha^{-1/2}. \qquad (3)$$

where h(s) denotes the dependence on the magnetic shear parameter $s=rq'/q$. The parallel wave number was also given as $k_{/\!/} = g(s)/qR$ (g(s) also indicates the s-dependence). Therefore, the ratio ω_k/ω_{te} for the relevant mode is given as

$$\omega_k/\omega_{te} \sim [h(s)/g(s)](qR/L_n)/\delta\sqrt{\alpha}. \qquad (4)$$

Noting that the relation $\delta^2 = 2\rho_e{}^2/\beta$ (the coefficient 2 comes from the assumption $T_e=T_i$) holds, we have

$$(\omega_k/\omega_{te})^2 \sim H(s)(qR/L_n)^2\beta/\alpha. \qquad (5)$$

where H(s) is defined as $[h(s)/g(s)]^2/2$. If we define the pressure-gradient scale length L_p by the relation

$$\beta'/\varepsilon = R\beta/L_p, \qquad (6)$$

Eq.(5) reduces to

$$(\omega_k/\omega_{te})^2 \sim H(s)RL_p/L_n{}^2. \qquad (7)$$

The right hand side of Eq.(7) is of the order unity for the typical plasma profiles. For example, for the shear parameter of $s=1$, the stability analysis gives $h(s)\sim 0.12$ and $1/g(s)\sim 1.6\pi$, yielding $H(s)\simeq 0.2$. In the case of the weaker shear, $s=0.5$, we have $h(s)\sim 0.31$ and $1/g(s)\sim 1.2\pi$, i.e., $H(s)\simeq 0.7$. In both cases, the ansatz of Ref.[8], $(\omega_k/\omega_{te})^2 << 1$, is disproved. Contrary to the conjecture of Reference [8], the ratio $(\omega_k/\omega_{te})^2$ is of order unity for the mode which is relevant to the calculation. It should be emphasized that the calculation in Ref.[8] ***confirmed*** that the expression of the current diffusivity, which we have used in the transport theory, is close to the kinetic counterpart.

The second claim was that we have ignored the off-diagonal components of the transport matrix. This was because the off-diagonal terms disappears when the the real frequency vanishes, which was the case in Refs.[1-4]. The fact, that the even- and odd-parity modes are mixed, has been well known in the kinetic study on tearing mode or drift mode[9]. The consistent solution of the real frequency and the transport coefficients is necessary, and is left for future research.

In summary, contrary to what was claimed in Ref.[8], the kinetic calculation of the current diffusivity gave the similar value to what we have obtained in our study on the self-sustained turbulence and anomalous transport. This would be one of the reasons that our result[1-4] explains very well the experimental results in tokamaks, in which the plasmas are almost collisionless.

We here notice the finite gyroradius effect. Using the estimate of k_θ, we have $(k_\theta \rho_i)^2 \sim (m_i/2m_e)h(s)^2(L_p/q^2R)$. In the framework of the theory, this could be a small parameter, since the theory was developed for the large-aspect-ratio limit, $a/R \to 0$. For the typical parameters, $s=1$, $q=3$ and $m_i/m_e=1836$, we have

$$(k_\theta \rho_i)^2 \sim 1.4 L_p/R \tag{8}$$

This result shows that the assumption of the small gyroradius correction is valid. However, in applying the theoretical result to the experimental data, the parameter $(k_\theta \rho_i)^2$ can become closer to unity. The finite gyro-radius effect may reduce the growth rate. On the other hand, it could reduce the ratio μ/λ and enhance the thermal transport. The future research is required.

It should be noted that the mode which is described by Eqs.(3) and (8) is the most-strongly-driven mode, and is not necessarily the mode of the largest amplitude. (Those with the largest amplitude would be observed as 'dominant component' in experiments.) As was pointed out in literatures, the E×B nonlinearity can cause the inverse cascade of the wave energy; the long-wave-length mode can have larger amplitude compared to the modes which are driven most strongly. (This nature was confirmed by numerical simulation[10].)

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